MECHANICAL METAMATERIALS

When size matters

That the unit cell of a metamaterial can't be considered vanishingly small like in ordinary crystals has long been deemed more burden than opportunity. The emergence of a characteristic length scale in metamaterial chains may change that trend.

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The concept of metamaterials has already blurred the boundary between a material's properties and its structures, architectures and mechanisms. But studies to date have focused on mechanical metamaterials that are scalable — in the sense that their effective properties do not depend on the number of unit cells. Now, writing in *Nature Physics*, Corentin Coulais and co-workers[1] have gone one step further to show that some metamaterials are characterized by a length scale, leading to an anomalous relationship between a material's properties and its size.

Using quasi-static experiments on one-dimensional chains comprising pairs of hinged ridged squares (Fig. 1), Coulais *et al.* showed that the effective moduli change with the number of unit cells along the chain. This behaviour indicates the presence of a characteristic length scale and leads to a non-monotonous dependence of the chain's Hookean spring constant on the number of unit cells with a superimposed even–odd oscillation. Usually the spring constant is simply inversely proportional to this number.

To bring their results into context, let us start by briefly revisiting ordinary materials under stationary conditions. The size of an atom or a crystalline unit cell is on the order of a few ångströms. This is more than a hundred million times smaller than your mobile phone. As a result, the unit cell can be seen as an infinitesimally small volume for most practical purposes. This view is the basis of textbook (that is, Cauchy) continuum mechanics[2] and has many important consequences. For example, the elastic parameters of the material, such as the compressibility or the shear modulus, do not depend on the overall material size. Clearly, the behaviour changes quantitatively — and possibly even qualitatively — if the number of atoms within the material decreases to mere thousands, hundreds or tens. However, this limit brings us away from materials science into the realm of macromolecular chemistry.

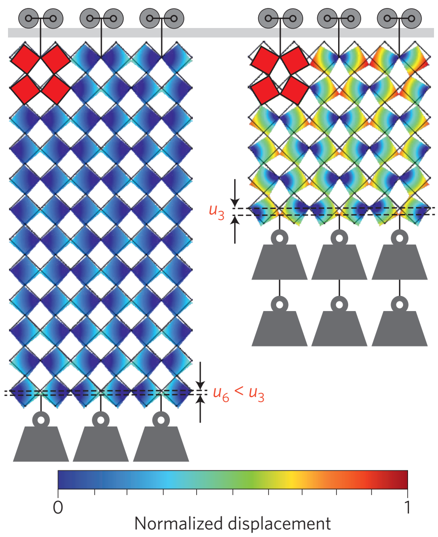

Figure 1 | Non-scalability in metamaterials. A unit cell composed of four hinged squares (highlighted in red) forms a two-dimensional mechanical metamaterial with N unit cells along the vertical direction and M along the horizontal. Shown are examples with $N = 6$ (left) and $N = 3$ (right) at fixed $M = 3$. The metamaterial samples hang on the ceiling and are subject to vertical gravitational forces via the weights at the bottom. If one bisects the length of an ordinary Cauchy-elastic material (not depicted) and doubles the masses at the bottom, the overall displacement u stays the same — and the material is scalable. The behaviour of the metamaterial is distinct: the displacement for six unit cells, u_6, is much smaller than that for three, u_3. This means that the metamaterial stiffness increases with N — and the metamaterial is not scalable. Scalability, however, is recovered for sufficiently large values of N, introducing a characteristic length scale.

In metamaterials, the unit cell of the metamaterial is man-made. It is custom designed and can have any size from nanometres to metres. Therefore, the unit cell can generally not be treated as being infinitesimally small. In some ways, this aspect was totally obvious from the start of the metamaterials field around 17 years ago[3,4]. However, it is usually considered a nuisance rather than an opportunity. The community has championed systems in which the effective material properties stay the same if the number of unit cells is changed — perhaps simply because the old habits of materials science die hard. In retrospect, this attitude has curbed additional design opportunities and the degrees of freedom that come about when considering metamaterials that are not scalable in this sense.

Through their experiments, Coulais *et al.* were able to show that this behaviour is connected to a characteristic length scale — in fact, they found two. For sizes much larger than this length scale, ordinary behaviour is recovered. This length scale is, of course, proportional to the size of the unit cell, but the pre-factor can be tailored by the architectural details of the unit cell. In their case, rotation-based deformations of the elements within the metamaterial unit cell determine this length scale.

Interestingly, they were able to derive an analytical expression for its scaling versus the geometrical parameters. It diverges if the connections between the squares can be considered ideal hinges. As the two hinged squares within their unit cell rotate in opposite directions, no overall (macroscopic) rotation of the chain results. All of these aspects are absent in ordinary Cauchy continuum mechanics, where the unit cell is treated as an infinitesimally small volume element.

The work raises a number of scientific questions and opens up avenues for further research. First, can one map the observed behaviour onto any kind of generalized effective-medium description? This question arises naturally when reading the extensive theoretical literature on micromorphic elasticity, summarized in the seminal textbook by A. Cemal Eringen[5]. Therein, the ordinary rank-four elasticity tensor is replaced by up to nine tensors. In addition to displacements, these tensors describe how rotations and deformations of the material microstructure — which can be periodic, but need not be — are connected to forces and torques. The tensor elements also directly determine the characteristic length scale.

This mapping onto generalized effective-medium parameters has, for example, been performed for human bone[6], suggesting that the effects of non-scalability are important in everyday life. It could well be successful for the present experiments too. Otherwise, a devil's advocate might argue that the properties Coulais *et al.* have observed are not effective metamaterial properties but rather properties of a complex structure made out of an ordinary elastic constituent material.

The study also prompts one to ask what the upper limits for the characteristic length scale might be in practice for any kind of mechanical metamaterial. This question is relevant because it would be even more striking if one could realize experimentally significant deviations from scalability in metamaterials with hundreds or thousands of unit cells — instead of order ten — along any one direction.

Finally, one wonders whether the lattice constants of millimetre order probed in the study could be drastically miniaturized to the microscale. To laymen, such microstructured metamaterials[7–9] would no longer seem to be mere toy models, but rather widely appreciated as real-world materials. ❐

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