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# Wave propelled ratchets and drifting rafts

A. EDDI[1(a)], D. TERWAGNE[2], E. FORT[3] and Y. COUDER[1]

[1] *Matières et Systèmes Complexes, Université Paris Diderot, CNRS - UMR 7057*
*Bâtiment Condorcet, 10 rue Alice Domon et Léonie Duquet 75205 Paris cedex 13, France, EU*
[2] *GRASP-Photopôle, Institut de Physique B5, Université de Liège - B-4000 Liège, Belgium, EU*
[3] *Matériaux et Phénomènes Quantiques, Université Paris Diderot, CNRS - UMR 7162*
*Bâtiment Condorcet, 10 rue Alice Domon et Léonie Duquet 75205 Paris cedex 13, France, EU*

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**Abstract** – Several droplets, bouncing on a vertically vibrated liquid bath, can form various types of bound states, their interaction being due to the waves emitted by their bouncing. Though they associate droplets which are individually motionless, we show that these bound states are self-propelled when the droplets are of uneven size. The driving force is linked to the assymetry of the emitted surface waves. The direction of this ratchet-like displacement can be reversed, by varying the amplitude of forcing. This direction reversal occurs when the bouncing of one of the drops becomes sub-harmonic. As a generalization, a larger number of bouncing droplets form crystalline rafts which are also shown to drift or rotate when assymetrical.



**Introduction.** – The self-propulsion of particles moving in a spatially periodic, assymetrical potential has been widely studied recently. Several physical systems have been proposed, aimed at providing models for the motion of biomolecular motors in which the energy input is the Brownian motion [1,2]. In these model systems an assymetry is needed which can have various origins. i) It can come from the substrate: it was shown, for instance, that Leidenfrost drops bouncing on an assymetrically structured medium translate spontaneously [3]. ii) It can be introduced in the moving object as, *e.g.*, when it is formed of an assymetrical assembly of two different structures [4]. iii) Finally, in non-linear systems the breaking of symmetry can be spontaneous and due to a bifurcation [5]. In a recent work [6,7], we showed that the latter phenomenon was responsible for the spontaneous horizontal displacement of a liquid drop on a vertically vibrated bath of the same fluid. In general the drop bounces at the forcing frequency and is otherwise motionless as described in [8–10]. Near the Faraday instability threshold, the bouncing becomes sub-harmonic and the drop becomes the source of a localized Faraday wave packet. By interaction with its own wave, the drop becomes a self-propelled "walker" moving on the surface at constant velocity [7]. In the present letter we investigate, in the same type of experiment, the behaviour of self-assembled assymetrical bound states. Each element is here individually motionless: the motion comes from the assymetry of their assembly.

**Experiment.** – The experiments are performed on a liquid bath of thickness $h_0 = 3\,\text{mm}$ submitted to a vertical oscillating acceleration $\gamma = \gamma_m \cos(2\pi f_0 t)$. In the following the control parameter of the system will be the non-dimensional amplitude of the forcing acceleration: $\Gamma = \gamma_m/g$. The liquid is silicon oil with viscosity $\mu_1 = 20 \times 10^{-3}\,\text{Pa}\cdot\text{s}$, surface tension $\sigma = 0.0209\,\text{N}\cdot\text{m}^{-1}$ and density $\rho = 0.965103\,\text{kg}\cdot\text{m}^{-3}$. The forcing frequency $f_0 = 80\,\text{Hz}$ is fixed. The drops are created by swiftly removing a pin dipped in the oscillating bath. The breaking of the liquid bridge between the pin and the bath can generate drops with diameters $0.1 < D < 1.5\,\text{mm}$. When the forcing amplitude is large enough, typically when $\gamma_m$ becomes larger than $g$, the drop lifts off at each period. The air film, which separates the drop from the substrate, is renewed at each period so that the bouncing can be sustained indefinitely [8,10]. The detail of their bouncing can be observed and recorded using a fast video camera (1000 images/s).

**Bound states.** – When two drops are present on the interface they "condense" into a stable bound state, a distance $d_{bd}^0$ separating them. The non-local interaction between drops is provided by the damped capillary waves

(a) E-mail: antonin.eddi@univ-paris-diderot.fr

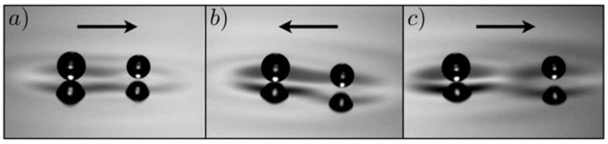


Fig. 1: Photographs of self-assembled bouncers forming self-propelled bound states. The two droplets have diameter of $D_1 = 0.73$ mm and $D_2 = 0.89$ mm, respectively. $\Gamma$ is the vertical acceleration normalized by $g$. In case a) ($\Gamma = 2.6 < \Gamma_1$), the two droplets are simple bouncers and the ratchet is pushed by the larger droplet. In case b) ($\Gamma_1 < \Gamma = 3.3 < \Gamma_2$), the smaller droplet has a period-doubled bouncing and has become the pusher. In case c) ($\Gamma_2 < \Gamma = 3.6$), the motion has reversed again as both drops have become period-doubled bouncers. See also the supplementary multimedia material Ratchet_motion.mov.

they emit. When a drop hits the bath, the collision forms a small crater in the surface. When the drop lifts up, the wave created by the shock evolves freely, the edge of the crater forming the crest of a circular wave propagating radially. In a bound state, each drop falls on a surface disturbed by the circular wave emitted during previous periods by its neighbour. With two drops of identical size, the system self-organizes in such a way that, during its collision with the bath, the horizontal impulse given to each drop is zero. This equilibrium can be obtained at a distance between drops $d^0_{bd} = \lambda_0 - \epsilon$, where $\lambda_0$ is the wavelength of the surface waves at the forcing frequency [7], and $\epsilon$ is an offset due to finite duration of the collision. With more than two drops the condensation leads to the formation of stable rafts with a crystalline lattice of the same periodicity $d^0_{bd}$. With drops of identical size both the bound states and the clusters are motionless. When they are formed of drops of different sizes, they have a spontaneous drift motion.

We first focus on the association of two drops of diameter $D_1$ and $D_2$ (with $D_1 < D_2$). For low values of $\Gamma = \gamma_m/g$ the bound state they form is observed to translate, the large drop pushing the small one. This is the mode 1 shown in fig. 1a. In fig. 2, we have plotted the velocity of several bound states as a function of $\Gamma$; mode 1 being associated to negative velocities. The bound state's velocity is a function of both diameters of the drops. Relatively fast translation motions are observed as, *e.g.*, $V = 3$ mm/s for drops with $D_1 = 1$ mm and $D_2 = 1.12$ mm.

When $\Gamma$ is increased, the velocity of the pair becomes at first larger, then a reversal of the direction of motion is observed so that the small drop now pushes the large one (mode 2, fig. 1b). This transition, observed for all pairs of drops, occurs at a value $\Gamma_1$ which is a function of the diameter $D_1$ of the smaller drop (fig. 2).

When the two droplets have diameters in between $D =$ 0.5 mm and 0.9 mm, a more complex sequence of behaviors is observed, characterized by two new thresholds, $\Gamma_2$ and $\Gamma_3$. Over the value $\Gamma_2$ there is a second reversal in the direction of motion (transition to mode 3, fig. 1c). In this case the large drop pushes again the small one. One can

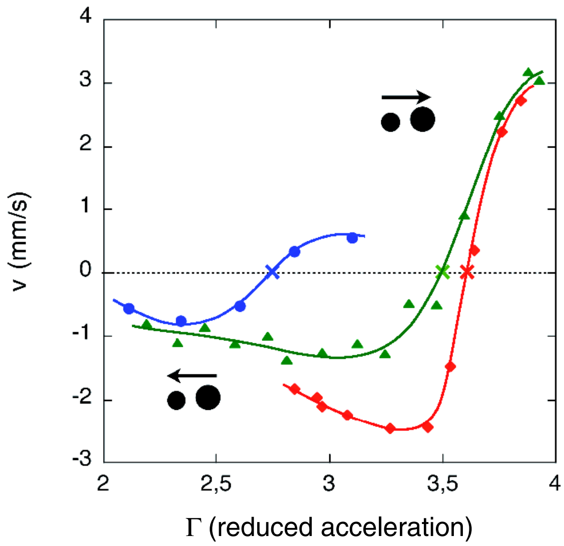


Fig. 2: The velocity of 3 couples of droplets as a function of $\Gamma$. The dots correspond to a couple of bound drops with $D_1 = 0.87$ mm and $D_2 = 0.96$ mm, the triangles to $D_1 = 1$ mm and $D_2 = 1.12$ mm and the rhombs to $D_1 = 1.03$ mm and $D_2 = 1.23$ mm. The continuous curves are simple interpolations. The crosses correspond to the reversal threshold $\Gamma_1$.

note that, after this transition, the distance $d_{bd}$ between the two drops becomes larger. Finally, the third threshold is reached at $\Gamma = \Gamma_3$, when the drops begin orbiting around each other.

Before giving an interpretation of these effects we must first characterize the bouncing of a single droplet. In our fixed experimental conditions (*i.e.* viscosity and forcing frequency being fixed) the types of observed bouncing are a function of the droplet diameter $D$ and of the reduced acceleration $\Gamma$ [7]. Figure 3a is a phase diagram which summarizes the behaviors observed for a single droplet. In the region B of this diagram, the drop bounces at the forcing frequency. When $\Gamma$ is increased the successive jumps become alternatively large and small, so that the period of the motion doubles (in region PDB of fig. 3a). The transition to this period doubling strongly depends on the drop's size. Larger drops do not lift away so easily because their deformation increases the size of their zone of near contact. Both the simple bouncing and the period doubling occur for larger values of $\Gamma$. This is related to the deformation of the drop during its collision with the substrate. This deformation depends on the drop's size $D$ since it is characterized by the Weber number: $We = (\rho V^2 D)/(2\sigma)$, the ratio of the kinetic energy of the drop to its surface energy. For drops of intermediate size, $0.5 < D < 0.9$ mm the period doubling can become complete so that the drop touches the surface once in two periods. Correlatively, it becomes a "walker" (in the region W in fig. 3a) moving at a constant velocity in the horizontal plane. The walkers were already investigated elsewhere [7].

We can now return to pairs of interacting drops. They form stable bound states at a well-determined distance

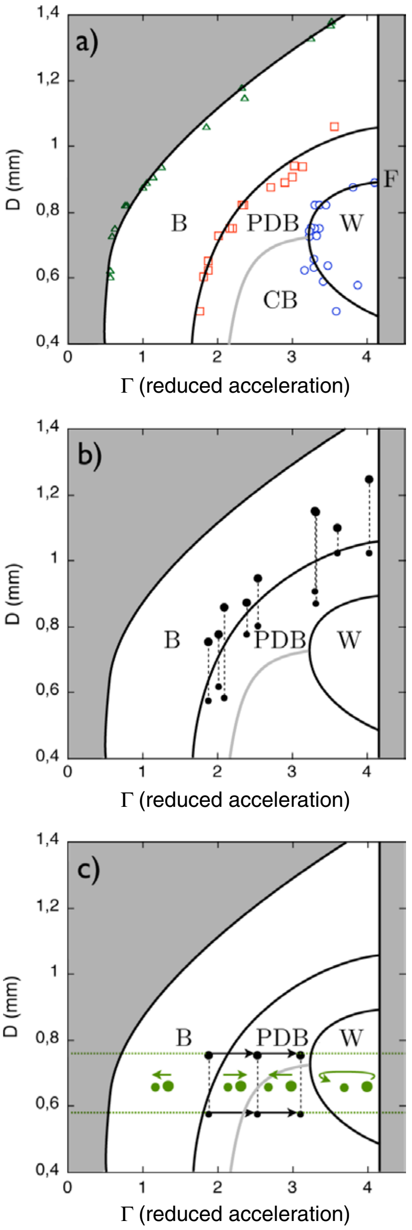


Fig. 3: a) Phase diagrams of the droplet's behaviour as a function of their diameter $D$ and of the reduced acceleration $\Gamma$. The viscosity of the silicon oil used is $\mu = 20\times10^{-3}\,\mathrm{Pa\cdot s}$ and the forcing frequency $f_0 = 80\,\mathrm{Hz}$. The various zones correspond to the different behaviors of a single droplet: in B it bounces at the forcing frequency, in PDB the bouncing has undergone a period doubling, in CB the bouncing has become chaotic through a period-doubling cascade. In region $W$ the droplets become walkers with a spontaneous horizontal displacement at a constant velocity. Region F is the Faraday instability zone. Panel b) shows, on the same phase diagram, the transition value $\Gamma_1$ for nine bound states. Two droplets forming a bound state are linked by a vertical dashed line. Their diameters are given along the ordinate axis. The abscissa at which they are represented is the value of $\Gamma_1$ for each bound state. Panel c) shows, with the same principle, the three thresholds observed for one single pair of droplets with diameters $D_1 = 0.57\,\mathrm{mm}$ and $D_2 = 0.75\,\mathrm{mm}$.

$d^0_{bd}$ from each other. Their non-local interaction is due to the surface wave they emit by their bouncing. The observed drift motion of uneven drops signifies that the forces exerted by the droplets on one another are not symmetrical: action appears to be different from reaction. In order to understand this drift we can first consider the motion of each droplet, then we will return to a global description of the system.

**Local behavior.** – The fast camera at a large magnification (see fig. 1) reveals images of the drops forming the ratchet in the three different regimes. In mode 1 both drops oscillate at the same frequency. However, the drops being unequal, neither the lift-off, nor the collision after the free flight, are simultaneous. At each period, the small drop hits the interface later than the large one. Being close to it, the small drop falls on the outer slope of the ridge of the crater formed by the larger drop. It thus receives a forward kick. Correlatively, the crater of the large drop becomes assymetrical by the collision of the small drop. As a result, at lift-off, the large drop also moves forward. Both drops thus receive a kick in the same direction and propagate together.

Where does the reversal in the motion direction come from? Just over the transition the small drop has undergone period doubling so that its successive collisions become uneven. One out of two shocks is weak and uneffective. During the other collision, the small drop hits the interface before the large one and repells it (see fig. 1b). This is confirmed by direct observation with a fast video camera. In fig. 3b, we have used the previous phase diagram, fig. 3a, of the individual droplets to represent the threshold values of $\Gamma_2$ for pairs of droplets. $\Gamma_2$ is systematically slightly larger than the value of $\Gamma$ for which the smaller drop has entered the region of period doubling.

**Global considerations.** – The two drops receive energy from the vibration generator but, as the imposed vibration is vertical, it does not directly provide a driving force. The motion is due to the initial asymmetry resulting from the different size of the droplets. This is natural: considered as a whole the two drops do not form an isolated system because they emit waves which propagate away. The waves are carrying away a flux of momentum which can be estimated. Observing the waves far from the bound state, one can assume that they are locally travelling plane sinusoidal waves with surface elevation $\zeta = a\cos(\omega t - kx)$, $a$ being the wave's amplitude, $\omega$ its pulsation and $k$ its wave number. Such waves possess an average momentum $1/2(\rho\omega a^2)$ per unit surface, with $\rho$ the fluid density [11,12].

This action reaction effect between the droplets and the waves explains the ratchet motion. However, it should be recalled that there is no exact momentum conservation in our system, because of dissipation. Consequently, it is not possible to make this argument more quantitative. Besides dissipation is needed. The emitted waves are damped by viscosity before reaching the boundaries so that no reflected wave returns to the ratchet. If dissipation vanished, waves would reflect on the borders and accumulate on the whole bath. The droplets would then have a chaotic motion on those waves. In a finite cell the breaking of time symmetry by dissipation thus appears necessary to propulsion. Note that in an infinite system dissipation should not be needed, causality being sufficient to give a direction to the wave propagation.

We can now consider the flux of momentum due to each of the droplets. At low forcing acceleration, observation with a fast camera shows that both drops oscillate at the forcing frequency $f_0$. Because of the difference of their masses the waves emitted by the two drops have different amplitudes, the larger the mass, the larger the amplitude. The reaction, resulting from the emission of momentum by the waves, pushes the bound state, the large droplet being behind.

With the increase of $\Gamma$, the bouncing of the smaller drop undergoes a period-doubling transition while the larger drop continues bouncing at the forcing frequency.

After period doubling, the small drop begins to emit surface waves of frequency $f_0/2$. This is the Faraday frequency, which is the least damped by the system, because of the proximity of the Faraday instability threshold. Independent measurements enable us to measure the amplitude of the waves. In the typical situation shown in fig. 4, the amplitude of the wave emitted by the small droplet is approximately five times greater than the amplitude of the wave emitted by the larger drop. Thus, the assymetry of the wave emission is reversed and the flux of momentum carried away by the emitted wave is larger on the side of the small drop. The resulting reaction pushes the bound state, the small drop being behind. Figure 3b confirms that $\Gamma_1$ corresponds to the value for the period doubling of the smaller drop.

When the two droplets have diameters in the narrow range between $D = 0.5$ mm and 0.9 mm, at a value $\Gamma_2$ the distance $d_{bd}$ between the two droplets increases and correlatively the direction of motion changes again. The plot in fig. 3c confirms that $\Gamma_2$ corresponds to the value for the period doubling of the larger drop. In this case, both droplets emit waves at the Faraday frequency. The distance $d_{bd}$ changes accordingly to reach $d^1_{bd} = \lambda_F - \epsilon$, where $\lambda_F$ is the wavelength of the surface waves at the Faraday frequency (see fig. 1c). Finally, the third threshold $\Gamma_3$ is reached where the drops begin orbiting. This corresponds to the situation when one of the drops achieves a complete period doubling and enters the W region of the phase diagram (see fig. 3c).

**Drifting and rotating aggregates.** – The existence of aggregates has been previously investigated [7,13,14]. However, we show here a new behaviour relying on the same physical effect as the spontaneous motion of a two-droplets bound state. When the drops forming the aggregate are of uneven size, a slow spontaneous displacement of the whole cluster is observed. The nature of the motion depends on the symmetry of the system. The global shape of the small agregates is dominated by the trend to form a triangular lattice. It is combined with a trend for an aggregate formed of a given number of particle to minimize its outer perimeter.

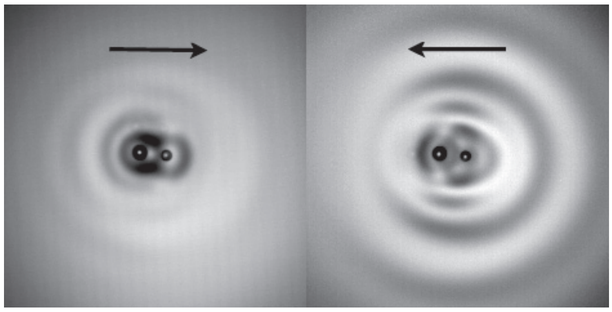

Fig. 4: Two top views of the same ratchet. Left: below $\Gamma_1$ the larger drop emits a wave of larger amplitude. Thus the bound state propagates to the right. Right: over $\Gamma_1$, the smaller droplet has undergone a period doubling and emits a Faraday wave of larger amplitude: the bound state propagates to the left.

The simplest possibility is the case of three drops. This type of bound state, with either a small drop and two large ones or the reverse situation, present the same first reversal. If we consider larger aggregates, in first approximation, each drop located at the periphery emits a wave, which can only propagate in the free surface. The reaction to the emission of the wave by each droplet placed at the periphery will be perpendicular to the local facet or if it is located at a vertex, along the bisector of the wedge under which it "sees" the free surface. If the direction of emission passes through the center of mass of the aggregate, the reaction will generate a drift, if not it will create an angular momentum and the cluster will rotate. Figure 5 shows two rotating aggregates. For drops which have diameter $D$ in between $D = 0.5$ and 0.9 mm, they are able to undergo a transition to a subharmonic bouncing. At the transition, the drops take various phases relatively to the forcing frequency. The mutual distance between two drops depends on whether they bounce in phase or with opposite phases. The aggregate thus becomes disordered before reorganizing in a more complex crystalline structure with two typical lengths that will be discussed elsewhere.

**Conclusion.** – In our experiment each bouncing drop is a mobile wave source. If isolated, it is either motionless or can move at a constant velocity by breaking of symmetry. When several wave emitters are present simultaneously on the surface they interact and form bound states and organized clusters. Here we have shown that these systems, when formed of uneven droplets, are spontaneously mobile by reaction to the waves they emit outwards. Such a mean of propulsion by reaction to the emission of surface waves was one of the mechanisms

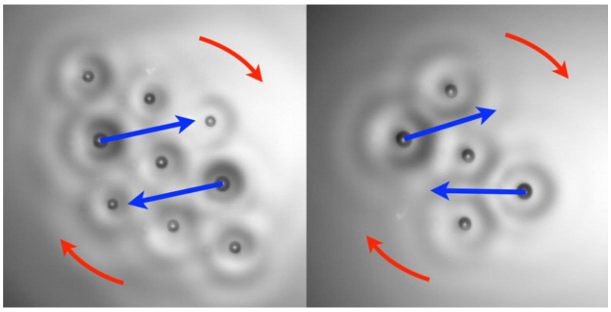

Fig. 5: (Top view) Two crystalline aggregates made of simple bouncers. In order to make the cluster asymmetric, all droplets are identical except two which have a larger diameter. In the two cases, the cluster rotates and the rotating period is around 90 s. The straight arrows represent the propulsion in reaction to the emission of surface waves.

proposed for the motion of water striders [15], even if more recent works [12] have shown that in this case the emission of vortices is the dominant effect. In our experiment, as the reaction depends on the frequency and the amplitude of the waves, bound states of uneven-size droplets present reversals in their motions when one of the drops undergoes subharmonic period doubling. For aggregates of uneven-size droplets, the asymmetry leads to spontaneous translation or rotation.

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