Metastable bouncing droplets

D. Terwagne, T. Gilet, N. Vandewalle, and S. Dorbolo
Department of Physics B5, GRASP-Photopôle, Université de Liège, B-4000 Liège, Belgium

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An oil droplet is placed on an oil bath at least ten times more viscous. When the oil bath is vertically shaken according to a sinusoidal motion, the droplet may bounce indefinitely on the bath. This sustained bouncing mode is stable when the forcing acceleration is higher than a threshold value. On the other hand, the time of subsidence of the droplet on the bath is finite at weaker accelerations. The lifetime of bouncing droplets is measured for various viscosities and forcing parameters. The finiteness of the lifetime is correlated with the evolution of the interference fringe pattern made by the air film squeezed between the droplet and the bath when lighted by a monochromatic light. Finally, a model based on a combination of lubrication theory and droplet deformations rationalizes the experimental results. © *2009 American Institute of Physics*. [DOI: 10.1063/1.3139138]

I. INTRODUCTION

In a remarkable set of experiments, Couder *et al.*[1] showed that droplets may bounce on a liquid bath that is vertically vibrated, a phenomenon that was observed for the first time in 1978 with vibrated soap solutions.[2] Considering a sinusoidal forcing motion of amplitude A and frequency f, Couder *et al.* evidenced that, as for an elastic ball, sustained bouncing is possible when the reduced acceleration of the forcing $\Gamma=4\pi^2Af^2/g$ is higher than a threshold value Γ_{th}, where g is the gravity. Conversely, when $\Gamma<\Gamma_{th}$, the droplet remains on the surface for a while before coalescing with the bath. The threshold Γ_{th} therefore represents the minimum forcing acceleration required to compensate for gravity.

A first attempt to model the permanent bouncing has been made by Couder *et al.*[1] More recently, this model has been significantly improved by taking the droplet deformation into account.[3,4] Permanent bouncing is shown to result from a combination of two factors. First, as shown by Reynolds,[5] a thin air layer remains between the droplet and the bath. This film, which prevents the droplet from coalescence, is squeezed at each bounce. When $\Gamma<\Gamma_{th}$, the lifetime of bouncing droplets is observed to be finite: the air film completely drains out in a finite time. Conversely, the air film should be constantly replenished in order to avoid coalescence when $\Gamma>\Gamma_{th}$. Second, the droplet is deformed at each bounce,[6] which may therefore induce some internal motions. As shown in Ref. 3, these latter are essential to explain the permanent bouncing since they deeply modify the drainage of the air layer. Moreover, due to viscous dissipation, these internal motions represent the main energy loss process.

The threshold acceleration is expected to depend on the forcing frequency f, the droplet mass M (which corresponds to a sphere of radius R and density ρ), and the physical properties of both liquids and the air layer (kinematic viscosity ν, surface tension σ, and density). Deformations are usually driven by surface tension and damped by viscosity effects. The Ohnesorge number $\mathrm{Oh}=\nu\sqrt{\rho/\sigma R}$, defined as the ratio between viscosity and surface tension, determines whether the deformation is significant or not. When $\mathrm{Oh}\ll 1$, stationary capillary waves are observed on the bouncing droplet, which may be described by spherical harmonics.[7,8]

In the literature, two kinds of systems have been considered up to now: homogeneous systems, where both the bath and the droplet are made with the same liquid, and heterogeneous systems, where the bath is made of a different liquid from the droplet. Quantitative modeling of the bouncing dynamics has not been achieved yet in the first case. Indeed, both deformations (bath+droplet) need to be taken into account. This difficulty is overcome in heterogeneous systems where viscosities may be chosen such as $\mathrm{Oh}\gg 1$ for the bath and $\mathrm{Oh}\leq 1$ for the droplet. In these conditions, the bath deformation may be neglected. In both homogeneous and heterogeneous systems, the droplet may use deformations for horizontal self-propulsion. In low viscosity homogeneous system, a single droplet may surf on its own waves.[9] An aggregate made of various droplet sizes can also lead to self-propulsion: the motion is induced by the asymmetry in the surface waves emitted by the aggregate on the bath. Moreover, the direction of motion can be controlled by increasing the forcing amplitude.[10] In heterogeneous system, droplets rely on their own deformation to roll and move on a flat bath surface.[8]

The lifetime of bouncing droplets have been investigated for homogeneous systems[11] above the threshold ($\Gamma>\Gamma_{th}$). The droplet motion is shown to be periodic (as well as the air film thickness evolution). Nevertheless, droplets with $\mathrm{Oh}\geq 1$ may bounce indefinitely while less viscous droplets experience a finite lifetime that depends on the viscosity, the forcing parameters, etc. Below Γ_{th}, no significant lifetime was observed: droplets instantaneously coalesce with the bath.

The situation is completely different in heterogeneous systems: the droplet may bounce during tens of seconds before coalescing on a vibrated bath with $\Gamma<\Gamma_{th}$. This paper reports an experimental investigation of this finite lifetime in heterogeneous systems. The droplet viscosity and forcing parameters (acceleration Γ and frequency f) are varied. Afterwards, the model developed in Ref. 3 is extended in order to rationalize the observations.

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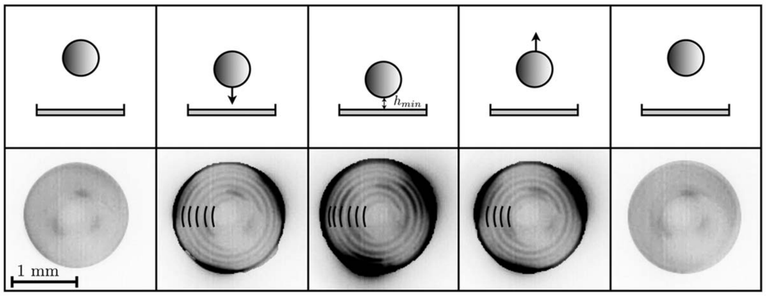

FIG. 1. Successive snapshots of interference fringes (accentuated by black arcs) made by a bouncing droplet. The snapshots are separated by 8 ms. The number of observed fringes is directly related to the film thickness which oscillates in between 100 nm and 10 μm. An increasing number of fringes are visible when the droplet lands and squeezes the air film (until snapshot 3); then this number decreases as the droplet takes off.

II. EXPERIMENTAL SETUP

A circular tank filled with 1000 cSt silicon oil (Dow Corning 200) is placed on an electromagnetic shaker (G&W 20). Oil droplets of constant radius $R=0.84$ mm are generated with a syringe (needle diameter of 0.35 mm) and are gently placed over the bath surface. The lifetime is measured starting from the first bounce on the bath. Three droplet viscosities have been tested: 10, 50, and 100 cSt, corresponding to Oh of about 0.074, 0.37, and 0.74, respectively. The Ohnesorge number based on the bath viscosity is about 7.4, which ensures that the bath deformation may be neglected. The acceleration Γ is tuned between 0 and $\Gamma_{th}(f)$ for frequencies between 25 and 100 Hz. In this range, the droplet deformation is accurately modeled by the spherical harmonic Y_2^0.[3] The droplet typically deforms by about 5%–10% of its radius. At least 18 droplets are made for each set of parameters. The standard deviation of the measured lifetime is reported as a typical error bar.

Information about the air film thickness may be obtained by using monochromatic lights (low pressure of sodium). Indeed, since the film thickness is typically micrometric, interference fringes are observed. According to Ref. 11 the number of fringes is related in a complex (but monotonic) way to the thickness of the film. These fringes are recorded from the top with a high-speed camera (Fig. 1). An aliasing technique is used to overcome the 50 Hz oscillation of the lamp.[11] Fringes appear when the droplet and the bath are in apparent contact and disappear when the droplet takes off. Note that the term "take off" is improper since the droplet and the bath are never in contact. A droplet is considered to take off when the lubrication film is sufficiently thick for the force it exerts to be neglected: interference fringes are no more visible and the droplet may be considered as in free fall (gravity is the only external force).

III. RESULTS

First of all, we have studied the statistical distribution of the lifetime when the forcing acceleration is smaller than the threshold Γ_{th}. In Fig. 2, the cumulative distribution function is plotted for $\Gamma=0.15\Gamma_{th}$ and $\Gamma=0.6\Gamma_{th}$, both with $f=75$ Hz and $\nu=10$ cSt. Note that the mean lifetime $\langle\tau\rangle$ is subtracted from measurements. Both cumulative distributions have been fitted by error functions as represented by the continuous lines in Fig. 2. We observe that the distribution of measured lifetimes is much narrower for the largest value of Γ.

In Fig. 3, the lifetime is plotted as a function of the forcing acceleration for various droplet viscosities and for $f=100$ Hz. The acceleration is normalized by the threshold acceleration Γ_{th}. The lifetime decreases and tends to 0 with increasing Γ then abruptly goes to very high values as Γ reaches Γ_{th}. The steepness of the curve clearly underlines that Γ_{th} represents a threshold for permanent bouncing. As seen on the error bars of Fig. 3 and confirmed by our statistical analysis (Fig. 2), the scattering of experimental data is overly important when $\Gamma \ll \Gamma_{th}$, while the lifetime seems more deterministic for larger values of Γ.

Comparing data from different viscosities, an increase in the droplet viscosity results in an increased lifetime. Measurements have also been made (but not reported here) with a 1.5 cSt droplet: the lifetime is less than 1 s when $\Gamma < \Gamma_{th}$. Nevertheless, it seems that the influence of viscosity cannot be caught by simple scaling arguments. Indeed, the lifetime at 50 cSt is closer from the 10 cSt curve than from the 100 cSt curve.

At $f=100$ Hz, the lifetime decay occurs for characteristic accelerations about $\Gamma \simeq 0.5\Gamma_{th}$, whatever the viscosity. On the other hand, we can see in Fig. 4 that the decay acceleration increases when the forcing frequency is decreased, shifting the whole lifetime curve to the right. This experimental

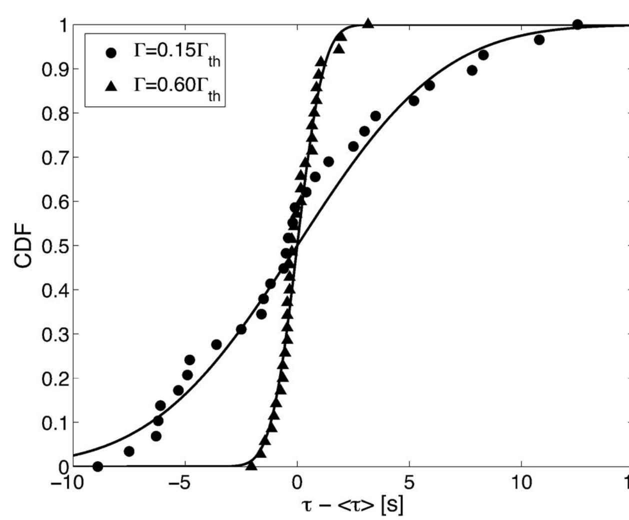

FIG. 2. Cumulative distribution F of the normalized lifetime of bouncing droplets ($f=75$ Hz, $\nu=10$ cSt) at various forcing accelerations: (i) $\Gamma/\Gamma_{th}=0.15$ (●) and (ii) $\Gamma/\Gamma_{th}=0.6$ (▲). The lines represent fitted Gaussian distributions.

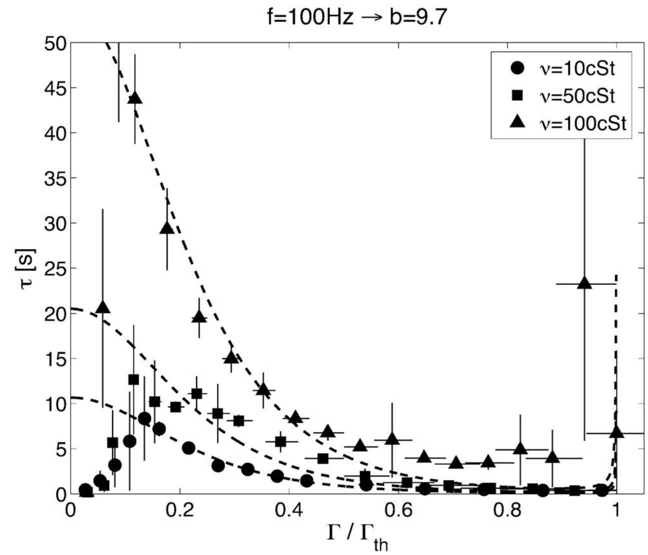

FIG. 3. Lifetime τ of bouncing droplets as a function of the normalized acceleration Γ/Γ_{th} for various droplet viscosities. (●) ν=10 cSt, (■) ν=50 cSt, and (▲) ν=100 cSt. Solutions of the model are represented by dashed curves. The frequency (f=100 Hz) corresponds to a value of b equal to 9.7.

fact, which cannot be explained by simple arguments, is rationalized by the model developed below.

In order to connect the lifetime to air drainage mechanisms, the air layer thickness has been observed, thanks to interference fringes for $\Gamma=0.22\Gamma_{th}$, f=100 Hz, and ν=100 cSt. In these conditions, the mean lifetime is about 20 s. The film thickness is estimated to oscillate in between 100 nm and 10 μm. In Fig. 5, fringes are shown for oscillating phases corresponding to the minimum in film thickness h_{min}. The number of fringes progressively decreases, which evidences that, even if the film is partially regenerated at each bounce, it slowly empties in average, in accordance with the Reynolds theory of lubrication.

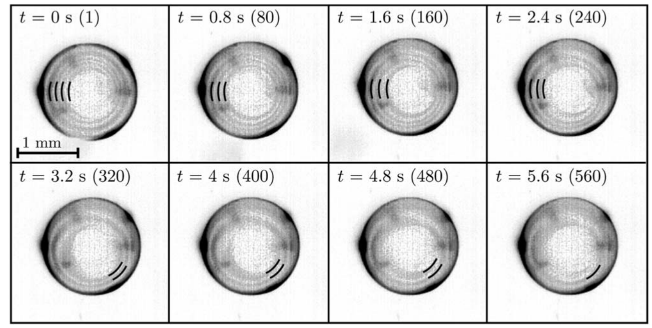

FIG. 5. Successive snapshots of the interference fringes (accentuated by black arcs) taken when the air film thickness is minimal for a droplet bouncing at f=100 Hz, $\Gamma=0.22\Gamma_{th}$, and ν=100 cSt. The number of fringes decreases with time, indicating a progressive decrease in the average film thickness. The number of bounce from the start is indicated in brackets on each snapshot.

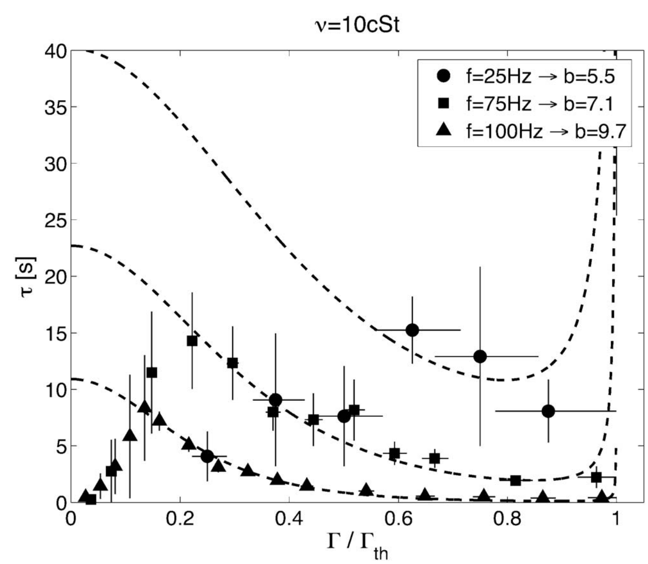

FIG. 4. Lifetime τ of bouncing droplets as a function of the normalized acceleration Γ/Γ_{th} for ν=10 cSt and various forcing frequencies. (●) f=25 Hz, b=5.5, (■) f=75 Hz, b=7.1, and (▲) f=100 Hz, b=9.7. Solutions of the model are represented by dashed curves.

IV. MODELING

According to measurements, both droplet viscosity and forcing parameters deeply modify the drainage kinetics (and consequently the lifetime). In this section, we extend the analytical model proposed in Ref. 3 in order to rationalize the observed lifetime variations. This model is based on two differential equations. The first one is a force balance on the droplet mass center in a frame of the bath,

$$M\frac{d^2x}{dt^2} = Mg(\Gamma \cos 2\pi f t - 1) + F, \quad (1)$$

where F is the lubrication force exerted by the squeezed air layer on the droplet. The second equation is the energy balance on the droplet in the frame of the droplet mass center,

$$\frac{d(K+E)}{dt} = -P_d - P_f, \quad (2)$$

where K is the kinetic energy of internal flows, E the potential surface energy, P_d the power dissipated by viscous effects, and P_f the power developed by F. Constitutive laws are needed in order to express those quantities in terms of two variables, namely, the vertical deformation η [defined by supposing that the droplet is a spheroid with vertical axis $(1+\eta)R$] and the film thickness $h=x-R(1+\eta)$. Constitutive laws proposed in Ref. 3 are $K=0.05M\dot{\eta}^2$, $E=5\sigma\eta^2$, $P_d=3.3\nu M\dot{\eta}^2/R^2$, and $P_f=\dot{\eta}F$. The lubrication force is estimated by considering the internal flow resulting from the droplet deformation,

$$F = \frac{3\pi}{2}\mu_a R^4\left(25\frac{\dot{\eta}}{h^2} - \frac{\dot{h}}{h^3}\right). \quad (3)$$

During bouncing, the droplet deformation is seen to vary by about 0.05, which correspond to internal flows of about 1 cm/s. The influence of those flows on the lubrication force, modeled by the term $\dot{\eta}/h^2$, is as important as the term $\dot{h}/h^3$ commonly encountered in the lubrication theory. Coefficients of the constitutive laws are obtained in Ref. 3 by fitting the model results on the experimental $\Gamma_{th}(f,\nu)$ curves. The coefficient of 25 has been changed to a slightly lower value, 17.5, which does not significantly affect the threshold curves fitting. Although this simple model is not expected to quantitatively reproduce the bouncing features (e.g., the droplet deformations, which are higher in the model), it provides a pretty accurate description of both the bouncing threshold and the lifetime, as it is seen below.

When the droplet does not bounce ($\Gamma < \Gamma_{th}$), the film remains thin and the droplet is no longer in free fall, so $\ddot{h} \ll g$. It is shown[3] that the evolution of the air film thickness $h(t)$ in the nonbouncing configuration is given by

$$h(t) = H(t)e^{17.5\text{Bo}\Gamma B(f)\cos(2\pi f t+\phi)}, \quad (4)$$

where the Bond number is defined as $\text{Bo}=Mg/\sigma R$, and

$$B(f) = \left[\left(10 - 4.4\pi^2\frac{Mf^2}{\sigma}\right)^2 + 43.6\pi^2\text{Oh}^2\frac{Mf^2}{\sigma}\right]^{-1/2}, \quad (5)$$

which expresses the amplitude of the droplet deformation varying with the frequency f.

The function $H(t)$ corresponds to the amplitude of the thickness variations, i.e., the thickness averaged over one bath oscillation. An approximation of $H(t)$ is derived in Ref. 3. Nevertheless, this approximation inevitably relies on the lubrication equation (3), which implies strong hypotheses about the droplet geometry and the internal motions. We assume a very general shape for $H(t)$,

$$\frac{1}{H(0)^2} - \frac{1}{H(t)^2} = \alpha C t, \quad (6)$$

where C is a function of the forcing parameters Γ and f, which satisfies $C(\Gamma=0)=1$ and $C(\Gamma=\Gamma_{th})=0$. Conversely, α depends on other parameters but not on these forcing parameters. In particular, α contains all the information derived from the lubrication theory for a droplet at rest on the bath (whatever the subjacent hypotheses) and depends on viscosities, geometry, etc. The function C obtained in Ref. 3 may be written as

$$C = I_0\left(b\frac{\Gamma}{\Gamma_{th}}\right) - \frac{\Gamma}{\Gamma_{th}}\frac{I_0(b)}{I_1(b)}I_1\left(b\frac{\Gamma}{\Gamma_{th}}\right), \quad (7)$$

where I_0 and I_1 are modified Bessel functions. The parameter b, which is proportional to $B(f)$ and $\Gamma_{th}(f)$, satisfies

$$\frac{I_0(b)}{bI_1(b)} = 37.5\text{Bo}\left(10 - 13.2\pi^2\frac{Mf^2}{\sigma}\right). \quad (8)$$

The monotonic function $b(f)$ is represented in the inset of Fig. 6. Assuming that the air film breaks at $t=\tau$ when reaching a critical thickness $H_C=H(\tau)$ gives

$$\frac{\tau_0}{\tau} = C\left[b(f), \frac{\Gamma}{\Gamma_{th}}\right], \quad (9)$$

where $\tau_0=\tau(\Gamma=0)$ is the lifetime that could be obtained without any forcing. This equation represents a family of curves $\tau/\tau_0(\Gamma)$ for various frequencies (Fig. 6). As seen in Figs. 3 and 4, those curves correctly fit the decreasing part of experimental data, the only fitting parameter being τ_0. The complex

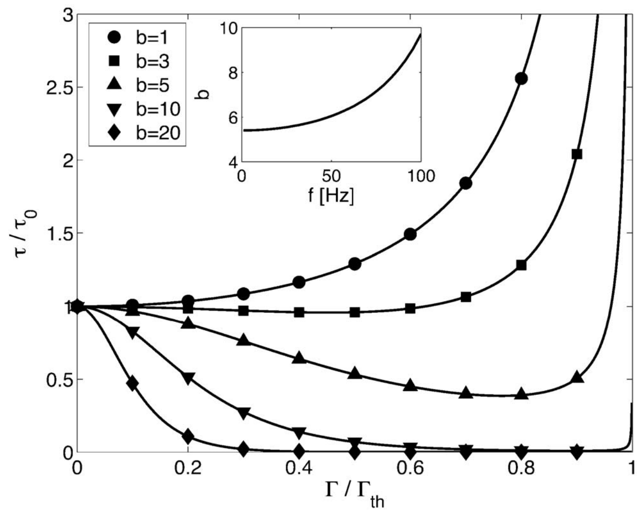

FIG. 6. Normalized lifetime τ/τ_0 as a function of b, according to Eq. (9). (Inset) Variation of b with the forcing frequency f.

variation of τ_0 with the droplet viscosity, mass, etc., is beyond the scope of this article. However, the model correctly catches the slow decreasing of the lifetime with an increase in Γ followed by a sharp divergence at the threshold acceleration Γ_{th}. As in experiments, the decay in $\tau(\Gamma)$ is right shifted when the frequency is increased. On the other hand, the increasing trend of $\tau(\Gamma)$ for very small values of Γ is not caught by the model. It is likely that other uncontrolled parameters (e.g., the residual flows created inside when the droplet is laid on the bath) considerably influence the lifetime at very low forcing accelerations.

V. CONCLUSION

When a high viscosity oil bath is vertically vibrated, oil droplets may bounce on it for a long time before coalescing. Permanent bouncing is ensured when the forcing acceleration Γ is higher than a threshold value Γ_{th}. Conversely, the lifetime τ of droplets is finite below this threshold. This particular regime has been explored in this paper. Observations of the interference fringes have confirmed that the lifetime finiteness is due to the slow drainage of the air layer between the droplet and the bath. The lifetime is shown to vary with the droplet viscosity and the forcing parameters (frequency f, acceleration Γ). In a large range of Γ, the lifetime slowly decreases with an increase in Γ. The $\tau(\Gamma)$ curve is shifted with variations of droplet viscosity ν and forcing frequency f.

A model based on two differential equations (Newton laws) allows to qualitatively reproduce the bouncing dynamics. It takes into account the air film drainage (lubrication theory) and the droplet deformation. Among others, this model catches the observed lifetime variations with forcing parameters Γ and f. With a single fitting parameter τ_0, the predicted lifetime correctly fits the experimental measurements.

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